SYNOPTIC: Development of the MHD Electrothermal Instability with Boundary Effects, A. H. Nelson, Imperial College of Science & Technology, London; *AIAA Journal*, Vol. 8, No. 10, pp. 1753–1759.

Plasma Dynamics and MHD; Electric Power Generation Research

Theme

This paper examines the stability of the nonequilibrium, partially ionized plasma associated with closed-cycle MHD generators. Using the Laplace transform technique, the time development of a small perturbation of a uniform plasma lying between two infinite walls is analyzed for the cases of insulator and finely segmented electrode walls.

Content

The electron density and temperature of a nonequilibrium partially ionized plasma (noble gas seeded with alkali metal) are described by the Saha equation and a time dependent energy equation. In addition, the current and electric fields are governed by the Ohm's law, Conservation of Charge, and Faraday's law. If $\mathbf{B}$, the magnetic field, is in the z direction and $\partial/\partial z = 0$ then the equations are

$$n_e^2/(n_s - n_e) = (2\pi m_e k T_e/h^2)^{3/2} e^{I_p/kT_e} \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_e k T_e + I_p n_e\right) = \frac{\mathbf{j}^2}{\sigma} - 3n_e k(T_e - T)\left(\nu_C \frac{m_e}{m_s} + \nu_B \frac{m_e}{m_n}\right) \tag{2}$$

$$\mathbf{j} = \sigma/(1 + \beta^2)(\mathbf{F} + \boldsymbol{\beta} \times \mathbf{F}), \mathbf{F} = \mathbf{E} + (1/n_e e)\nabla p_e \text{ and } \boldsymbol{\beta} = \mathbf{B}e/m\nu \tag{3}$$

$$\nabla \cdot \mathbf{j} = 0 \tag{4}$$

$$\nabla \times \mathbf{E} = 0 \tag{5}$$

where n_e, n_s = electron and seed density, respectively; T_e, T = electron and gas temperature, respectively; I_p = ionization potential of the seed; ν_C, ν_B = coulomb and neutral collision frequencies, respectively; m_e, m_s, m_n = electron, seed, and noble gas masses, respectively: $\mathbf{j}$ = current density, $\mathbf{E}$ = electric field, σ = conductivity, β = Hall parameter, and p_e = electron pressure.

It is well known from infinite plasma-plane wave analyses that the plasma is unstable to the electrothermal instability if β is greater than some critical value of the order of one. However the effect of the apparatus walls on the plasma stability is not known.

Using Eqs. (4) and (5) we can write

$$\mathbf{j} = \nabla \times \boldsymbol{\psi}, \mathbf{E} = -\nabla\phi, \boldsymbol{\psi} = (0,0,\psi)$$

and the boundary conditions at the walls are: a) ψ = constant on insulators, b) ϕ = constant on electrodes, and c) if electrode $\epsilon 1$ and electrode $\epsilon 2$ are connected externally by a load R_L, then

$$R_L \int_{\epsilon 1} \mathbf{j} \cdot d\mathbf{s} = -R_L \int_{\epsilon 2} \mathbf{j} \cdot d\mathbf{s} = \phi_{\epsilon 2} - \phi_{\epsilon 1}$$

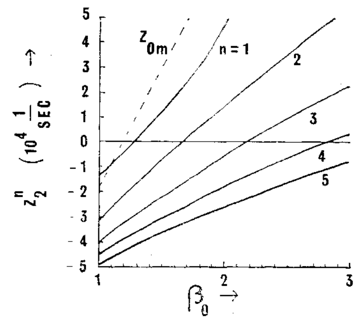

Fig 1 $\Re$ (Z_2^n) **vs Hall parameter (continuous insulator walls).**

Assuming that we have a uniform plasma lying between two walls parallel to the y-z plane, and we consider a Fourier component $e^{-IK_y y}$ of a general perturbation of n_e, then the linearized equations describing the time development of this component can be written in the form

$$A(\partial\xi/\partial t) + B(\partial\xi/\partial x) = 0 \tag{6}$$

where ξ is the transpose of the perturbation vector (n_e^*, ψ', ϕ') (note: $n_e^* = n_e'/n_{eo}$, n_{eo} being the value of n_e in the uniform plasma) and A and B are matrices which are functions of K_y and the uniform state.

Equation (6) is solved under the boundary conditions a, b, and c using the Laplace transform technique. We find that if $\bar{\xi}$ is the solution of the transform of Eq. (6) then

$$\bar{\xi} \propto 1/\det P$$

where P is a matrix incorporating K_y, the uniform state, and the boundary conditions. To obtain the solution $\xi(x,y,t)$ we have to inverse transform $\bar{\xi}$ and this leads to a series of terms of the form $\exp(Z_{\text{pole}}t)$ where Z_{pole} represents the poles of $\bar{\xi}(Z)$ (Z is the transformation parameter). The dispersion relation is therefore given by

$$\det P(Z) = 0$$

and the stability analysis is reduced to finding the roots of this equation with positive real parts.

In general, there exists an infinite number of roots, corresponding to an infinite number of modes. However, usually only a finite number of modes are unstable and this number increases with increasing Hall parameter [see Fig. 1, where n is the mode parameter, and $\Re(Z_{om})$ is the maximum growth rate from an infinite plasma theory]. The modes are either plane waves or approximately plane waves and each has a different orientation with respect to the walls. The successive mode destabilization therefore explains the experimentally observed transition from near plane wave to turbulent structure with increasing Hall parameter. Computer plots of the structure of the modes are presented in the paper.